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LAGEOS-type Satellites in Critical Supplementary Orbit Configuration and the Lense–Thirring Effect Detection

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Abstract

In this paper we analyze quantitatively the concept of LAGEOS-type satellites in critical supplementary orbit configuration (CSOC) which has proven capable of yielding various observables for many tests of General Relativity in the terrestrial gravitational field, with particular emphasis on the measurement of the Lense–Thirring effect. By using an entirely new pair of LAGEOS-type satellites in identical, supplementary orbits with, e.g., semimajor axes $a = 12000$ km, eccentricity $e = 0.05$ and inclinations $i_{S1} = 63.4^\circ$ and $i_{S2} = 116.6^\circ$, it would be possible to cancel out the impact of the mis-modelling of the static part of the gravitational field of the Earth to a very high level of accuracy. The departures from the ideal supplementary orbital configuration due to the orbital injection errors would yield systematic gravitational errors of the order of few percent, according to the covariance matrix of the EGM96 gravity model up to degree $l = 20$. However, the forthcoming, new gravity models from the CHAMP and GRACE missions should greatly improve the situation. So, it should be possible to measure the gravitomagnetic shifts of the sum of their nodes $\Sigma\dot{\Omega}^{LT}$ with an accuracy level perhaps less than 1%, of the difference of their perigees $\Delta\dot{\omega}^{LT}$ with an accuracy level of 5% and of $\dot{X}^{LT} \equiv \Sigma\dot{\Omega}^{LT} - \Delta\dot{\omega}^{LT}$ with an accuracy level of 2.8%. Such results, which are due to the non-gravitational perturbations mismodelling, have been obtained for an observational time span of about 6 years and could be further improved by fitting and removing from the analyzed time series the major time-varying perturbations which have known periodicities.

1 Introduction

The idea of using a pair of twin satellites, denoted as S1 and S2, in identical orbits with the same semimajor axes a and eccentricities e , except for the inclinations i of their orbital planes, which should be supplementary, in order to measure the general relativistic Lense–Thirring effect (Lense and Thirring 1918) in the gravitational field of the Earth¹ was put forth for the first time by Ciufolini with the proposed LAGEOS–LAGEOS III mission (Ciufolini 1986). The proposed observable is the sum of the rates of the longitudes of the ascending nodes

$$\Sigma\dot{\Omega} \equiv \dot{\Omega}_{S1} + \dot{\Omega}_{S2}. \quad (1)$$

Indeed, it turns out that while the Lense–Thirring secular nodal rates are independent of the inclinations of the satellites and add up in eq. (1), the classical secular nodal rates induced by the oblateness of the Earth, which would mask the relativistic effect due to the uncertainties in the even zonal coefficients $\delta J_2, \delta J_4, \delta J_6, \dots$ of the multipolar expansion of the terrestrial gravitational field, are equal and opposite for supplementary orbital planes because they depend on odd powers of $\cos i$, so that they would be cancelled out by eq. (1). Later on, the orbital and physical configuration of LAGEOS III slightly changed: the eccentricity of its orbit was increased in order to be able to perform other general relativistic tests, its mass was reduced so to reduce the mission–launch costs, and the area was reduced in such a way to guarantee the same area–to–mass ratio of the older LAGEOS, so to reduce the impact of the non–gravitational perturbations. Thus LARES was born (Ciufolini 1998). In Table 1 we quote the orbital parameters of some existing or proposed laser–ranged satellites which are used, or could be used, in general relativistic tests. The accuracy available with the originally proposed version of the LAGEOS–LARES mission should amount to 2%–3% (Ciufolini 1998). Very recently, some modifications of the observable to be adopted in the LARES mission have been suggested (Iorio *et al* 2002). The total error should then become $\sim 1\%$.

The concept of satellites in identical and supplementary orbits have been recently extended also to the perigees (Iorio 2002; 2003). In particular, it has been noticed that also the difference

¹In (Ciufolini *et al* 1998) an experimental check of such prediction of General Relativity in the field of the Earth by using the laser data of LAGEOS and LAGEOS II satellites is reported. The claimed accuracy is of the order of 20%.

Table 1: Orbital parameters of LAGEOS, LAGEOS II, LARES, S1 and S2.

Orbital parameter	LAGEOS	LAGEOS II	LARES	S1	S2
a (km)	12270	12163	12270	12000	12000
e	0.0045	0.014	0.04	0.05	0.05
i (deg)	110	52.65	70	63.4	116.6

of the rates of the perigees

$$\Delta\dot{\omega} \equiv \dot{\omega}_{S1} - \dot{\omega}_{S2} \quad (2)$$

could be considered, in principle, for measuring the gravitomagnetic field of the Earth. Indeed, the Lense–Thirring secular apsidal rates depend on $\cos i$ and add up in eq. (2), while the classical secular apsidal rates due to the oblateness of the Earth, which depend on $\cos^2 i$ and on even powers of $\sin i$, are equal and cancel out in eq. (2). Of course, such an observable could not be considered for the LAGEOS–LARES mission since the eccentricity of LAGEOS is too small and the perigee of its orbit is badly defined. On the contrary, launching an entirely new pair of LAGEOS–type satellites in rather eccentric orbits would allow to adopt both eq. (1) and eq. (2) and also

$$\dot{X} \equiv \Sigma\dot{\Omega} - \Delta\dot{\omega}. \quad (3)$$

In (Iorio 2002) it has been noticed that it should be better to adopt the critical inclinations $i_{S1} = 63.4^\circ$ and $i_{S2} = 116.6^\circ$ because, in this way, the periods of many time–dependent harmonic orbital perturbations of gravitational and non–gravitational origin would be not too long. So, it would be possible to adopt an observational time span T_{obs} of a few years. This fact would be important not only from the point of view of reducing the data analysis time, but also because certain relevant and very useful assumptions on the surface properties of the satellites and on their spins motion, which would affect certain subtle but important non–gravitational perturbations, could be safely done by adopting just the first years of life of both satellites for the data analysis.

In this paper we wish to analyze quantitatively the impact of many systematic errors induced by gravitational and non–gravitational perturbations on the proposed observables so to yield realistic error budgets for such new proposed gravitomagnetic experiments and clarify if the

alternative proposed observables are really competitive with the sum of the nodes.

The paper is organized as follows. In section 2 we will deal with the systematic error due to the mismodelling in the even zonal harmonics of geopotential and its sensitivity to the orbital injection errors in the inclinations of the satellites. In section 3 we will focus our attention on the impact of the non-gravitational perturbations. Section 4 is devoted to the conclusions.

2 The gravitational errors

Contrary to the originally proposed LAGEOS–LARES mission (Iorio *et al.*, 2002), with the new satellites S1 and S2 of Table 1, which have $a_{S1} = a_{S2}$, $e_{S1} = e_{S2}$, it would be possible to cancel out exactly from eqs. (1)–(3) the systematic errors due to the even zonal harmonics of the geopotential, provided that the inclinations of the two orbital planes are exactly supplementary.

Of course, this could not occur in reality because of the unavoidable orbital injection errors in the Keplerian orbital elements of the satellites, in particular those in the inclinations. In Figure 1–Figure 3 we show the impact of the deviations of the inclinations from the nominal condition of supplementarity on the systematic errors in $\Sigma\dot{\Omega}$, $\Delta\dot{\omega}$ and \dot{X} . The covariance matrix of the even zonal harmonics of the EGM96 Earth gravity model (Lemoine *et al.*, 1998) up to degree $l = 20$ has been employed². It is worth noticing that the results obtained here can be considered reliable even if the higher degree terms of EGM96 would not be accurate enough because the orbits of the LAGEOS–type satellites are almost insensitive to the harmonics of degree higher than $l = 20$. As it can be noticed, for deviations of the order of 1 degree³ from the nominal supplementary values, the systematic error due to the geopotential would amount to few percent. However, it is really important to notice that such estimates should be dramatically improved by the new, more accurate terrestrial gravity models from CHAMP and

²However, it should be mentioned that, according to (Ries *et al.*, 1998), it would be unjustified to extend the validity of the covariance matrix of EGM96 to any generic time span of a few years due to secular, seasonal and stochastic variations of the terrestrial gravitational field which have not been included in the solution of EGM96. Then, the validity of its covariance matrix should be limited just to the multi-decadal time span during which the data for its construction have been collected.

³In general, the orbital injection errors depend strongly upon the final stage of the rocket. Indeed, if it uses a solid propellant a conservative estimate yields $\delta i_{inj} \sim 1^\circ$, while with a liquid propellant, which is more expensive, $\delta i_{inj} \sim 0.6^\circ$ (L Anselmo, private communication, 2002). For the LAGEOS III project the predicted orbital injection error in the inclination, by using a McDonnell–Douglas launcher, was estimated to be 0.03° at 3- σ (See the McDonnell–Douglas document VDD5318 M2V, March 1987, cited in (Ciufolini, 1989)).

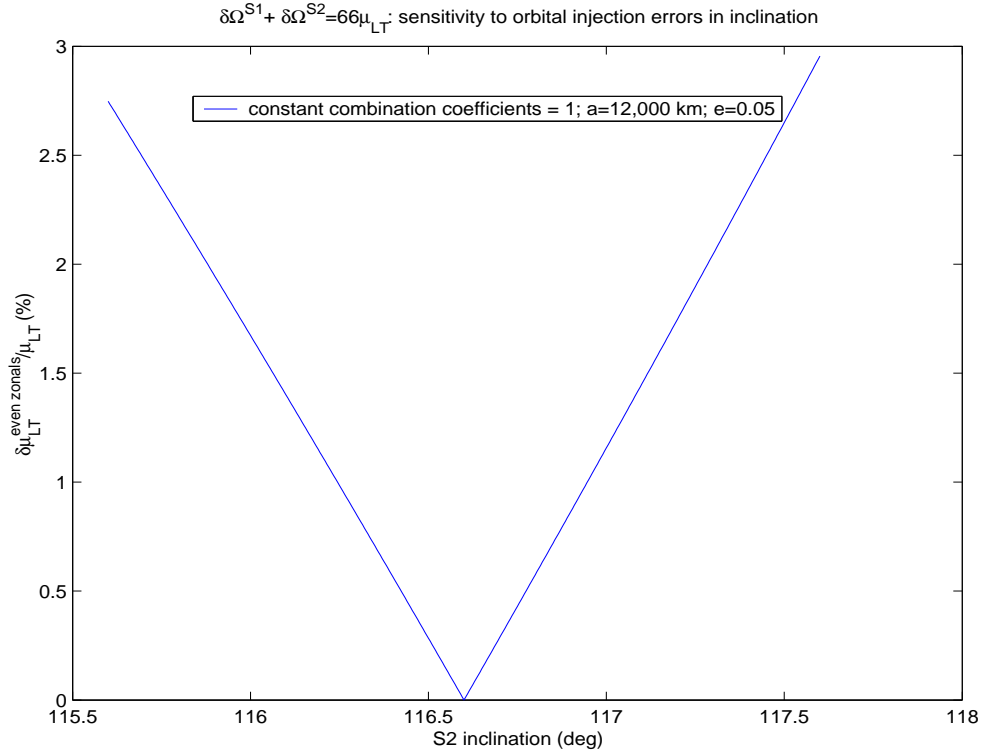


Figure 1: Influence of the injection errors in the inclination on the zonal error of $\Sigma\dot{\Omega}$.

GRACE missions. Even without claiming any precise predictions which could turn out to be too optimistic when the real data will be finally available, it should be realistic to expect that the systematic error due to the geopotential will fall well below the 1% level for all the observables considered here. It turns out that for deviations of a few kilometers of the semimajor axes from their nominal values the error due to the mismodelled even zonal harmonics of the geopotential is well below 1% for the three proposed observables.

In regard to the time-dependent gravitational perturbations of tidal origin (Iorio 2001), the perigees would be particularly sensitive to them. However, as pointed out in (Iorio 2002), on one hand, some very insidious tidal perturbations like that induced by the 18-6 year constituent are cancelled out by the difference of the perigees and, on the other, the choice of the critical inclination for the two satellites would allow to reduce greatly the periods of certain uncanceled perturbations which, then, could be fitted and removed over a T_{obs} of just a few years during which they would be able to describe some full cycles. It turns out that also the time-dependent

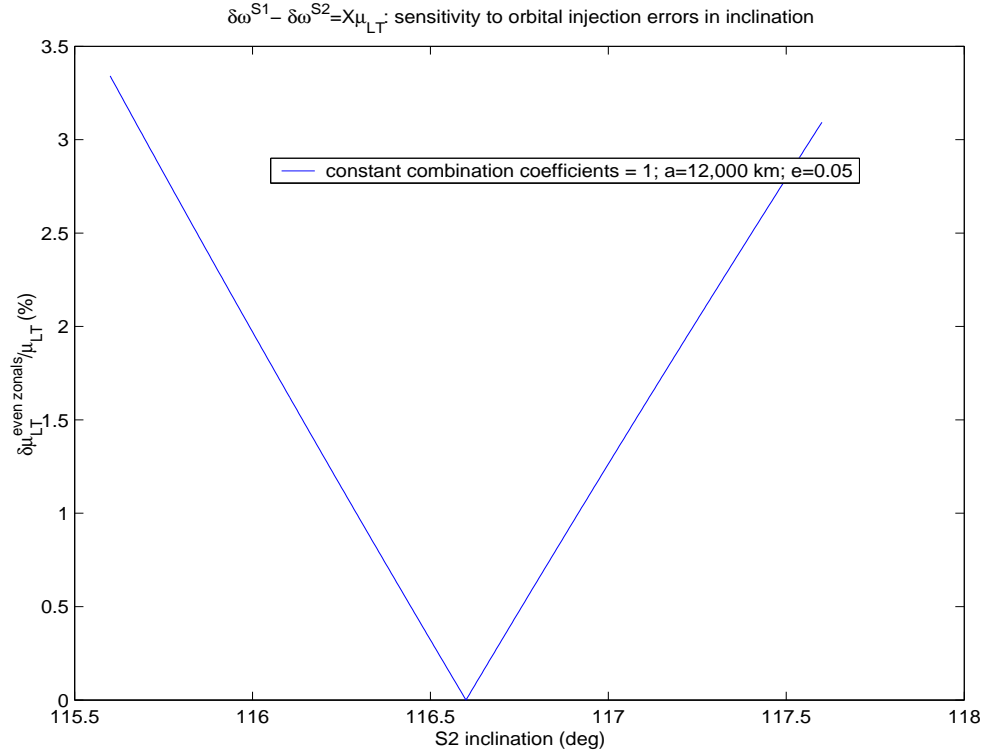


Figure 2: Influence of the injection errors in the inclination on the zonal error of the $\Delta\dot{\omega}$.

perturbations induced on the perigee by the J_{2n+1} odd zonal harmonics of the geopotential cancel out in the difference of the perigees. Moreover, it should be considered that also the uncertainty in the time-varying part of the terrestrial gravitational field would be reduced by the forthcoming models from CHAMP and GRACE, as can be found in <http://op.gfz-de/grace/>.

3 The non-gravitational perturbations

In view of the forthcoming improvements in the accuracy of our knowledge of the terrestrial gravitational field, a major role in the error budget of the proposed experiments will be played by the perturbations due to the non-conservative forces. Indeed, concerning the measurement of the Lense-Thirring effect using the existing LAGEOS satellites, we know that, according to (Lucchesi 2001; 2002a), a crucial role in the definition of the error budget is played by the mismodelling in the non-gravitational perturbations (NGP). The perturbative effects due to the visible radiation effects, as well as those related with the thermal thrust perturbations—if

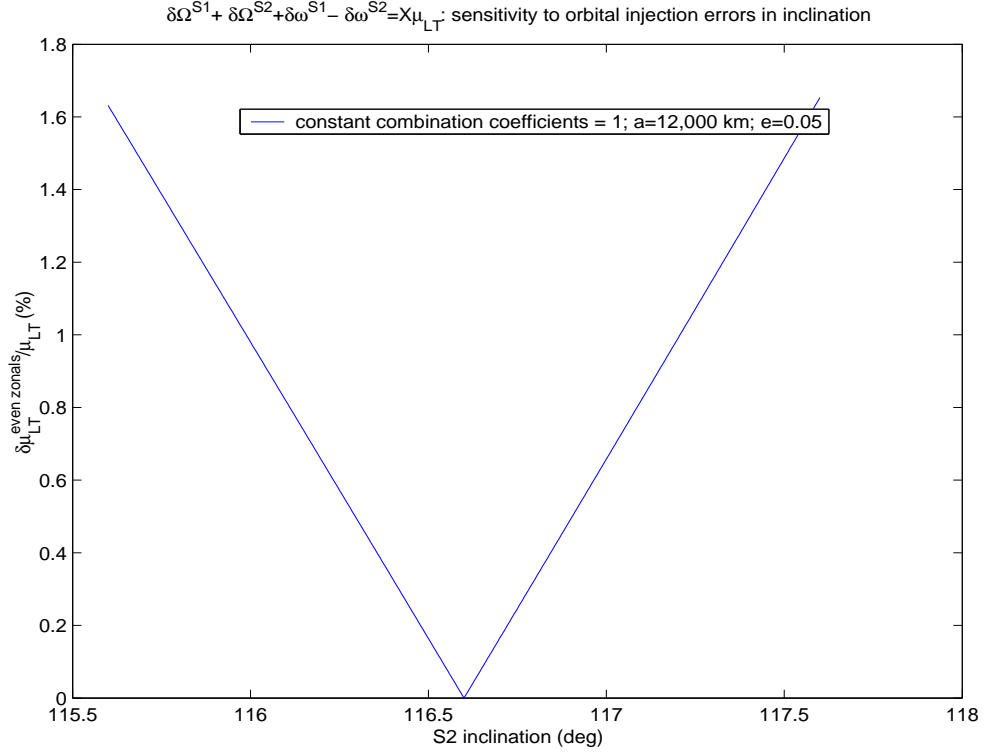


Figure 3: Influence of the injection errors in the inclination on the zonal error of \dot{X} .

not correctly modelled—may cause errors in the observable orbital elements comparable to or even larger than the relativistic secular shift. We computed, through a numerical simulation and analysis (considering the eclipses passages), the perturbative effects due to these non-conservative forces on the proposed combinations. Then, using the uncertainties of each perturbative model adopted we estimated, in a conservative way, the error budget due to these perturbations in the perigee and nodal rates of the proposed satellites and in each of their combinations. We analyzed the perturbative effects due to

- direct visible solar radiation,
- Earth albedo radiation,
- terrestrial Yarkovsky–Rubincam radiation,
- solar Yarkovsky–Schach radiation,
- possible asymmetric reflectivity

Only the terrestrial Yarkovsky–Rubincam thermal thrust perturbation produce secular effects in the satellites node and perigee. Nevertheless these secular effects are usually small—about 1 mas yr^{-1} or less—and negligible when compared to the long-term effects produced in the same elements by the other non-gravitational perturbations (Lucchesi 2002a). Then, in order to minimize the impact of these long-term effects, we estimated an *Ideal Period* (IP) during which a large fraction of the analyzed perturbations averages out. For this IP, tuned by the larger perturbative effects, we obtained a value close to 6 years (2187 days). This IP has been estimated from the longest periodicity, in the satellites perigee, due to direct solar radiation pressure—the largest perturbative effect—corresponding to the spectral line $\dot{\Omega} + \dot{\lambda} - \dot{\omega}$ (729-day) of the S1 satellite. The period obtained is a multiple integer of the other characteristics periodicities of the analyzed NGP, and is very close to the period of 734-day due to the node precession of the CSOC satellites, see sub-section 3.1. This IP of about 6-year represents a good compromise in the averaging out process of the non-gravitational perturbations and, at the same time, on the accumulation of the integrated orbital residuals over a time long enough to detect a so tiny effect as the Lense–Thirring dragging. Anyway, for a first measurement of the relativistic effect with the proposed CSOC satellites, a minimum IP of about 2 years (729 days) may be used. Of course, for a fixed IP, the averaging of the NGP depends also from the initial conditions of the satellites configuration, e.g., from the orientation of their orbits with respect to the Sun. This aspect has been tested performing several simulations over our 2187 days IP, but with different values for the initial conditions of the satellites orbit in space—in the ascending node and perigee initial positions—and for different periods of the year, i.e., of the Earth position along its orbit with respect to the Sun. In the following we present the results we obtained for an initial configuration (the same for the two proposed satellites) very close to that of LAGEOS when launched in May 1976. The results we obtained for the NGP mismodelling impact on the suggested combinations of the satellites node and perigee, may be considered a conservative approach to the final error budget. Indeed, we have been able to obtain a better averaging of the NGP effects using different configurations, but we also obtained larger effects in the perigee and node rates for other initial conditions. At the same time the proposed configuration for the two satellites has the advantage to be realized without particular difficulties using the actual launchers.

In regard to the Asymmetric Reflectivity effect we have made the assumption that the asymmetry in the reflection from the satellites surface is the same for the two satellites, and with the same value as the one estimated in the case of LAGEOS. Recently, in the case of LAGEOS II (Lucchesi 2002b), has been estimated that a large fraction of this asymmetry in the albedoes of the two hemisphere, could be explained by the specular reflection of the visible solar-radiation from the Germanium Cube-Corner-Retroreflectors (CCR) embedded in the satellite surface. If this would be correct, we can probably neglect the effect by not including these CCR on the satellite surface. But the problem of a complete physical explanation of the anisotropy in the case of LAGEOS satellites is still open, and possible other mechanisms may contribute to the asymmetry in addition to the Germanium CCR specular reflection of sun light. We then estimated the effects of this perturbation using the empirical values obtained in the case of LAGEOS and LAGEOS II (Lucchesi, 2002a). We can look again to our final error budget as a conservative estimate of the non-gravitational perturbative effects uncertainties to the proposed derivation of the Lense-Thirring precession on the combination of perigee and node of the CSOC satellites.

The rest of the section is organized as follows. In sub-section 3.1 we shall describe some of the details of the analytical results on the combinations of the proposed orbital elements of the two CSOC satellites. In sub-section 3.2 the results of a numerical simulation and analysis are given and briefly compared with the analytical results.

3.1 Analytical results

The relativistic Lense-Thirring precessions on the node and the perigee of the CSOC satellites are

$$\dot{\Omega}^{\text{LT}} = \frac{2G}{c^2 a^3} \frac{J_{\oplus}}{(1 - e^2)^{3/2}} = 32.9 \text{ mas yr}^{-1} \quad (4)$$

for both satellites, and

$$\dot{\omega}^{\text{LT}} = -\frac{6G}{c^2 a^3} \frac{J_{\oplus}}{(1 - e^2)^{3/2}} \cos i = \begin{cases} -44.2 \text{ mas yr}^{-1} & \text{in the case of S1} \\ +44.2 \text{ mas yr}^{-1} & \text{in the case of S2} \end{cases} \quad (5)$$

where G is the Newtonian gravitational constant, c is the speed of light in vacuum and J_{\oplus} is the Earth's angular momentum. Then, for the combinations proposed in this work we obtain

$$\Sigma \dot{\Omega}^{\text{LT}} \equiv \dot{\Omega}_{\text{S1}}^{\text{LT}} + \dot{\Omega}_{\text{S2}}^{\text{LT}} = 65.8 \text{ mas yr}^{-1} \quad (6)$$

$$\Delta\dot{\omega}^{\text{LT}} \equiv \dot{\omega}_{\text{S1}}^{\text{LT}} - \dot{\omega}_{\text{S2}}^{\text{LT}} = -88.4 \text{ mas yr}^{-1} \quad (7)$$

$$\dot{X}^{\text{LT}} \equiv \Sigma\dot{\Omega}^{\text{LT}} - \Delta\dot{\omega}^{\text{LT}} = 154.2 \text{ mas yr}^{-1} \quad (8)$$

We will focus on the perigee rate of the proposed satellites because of the larger perturbations and less accurate determination of this element with respect to the node. The effect of the direct solar radiation pressure on the perigee rate for a spherically shaped, passive, laser-ranged satellite of LAGEOS-type is

$$\dot{\omega} = \frac{3a_{\odot}}{8nae} \left\{ \begin{aligned} & \left[1 - \left(1 + \cos i \right) \cos \epsilon + \cos i \right] \cos \left(\Omega + \lambda + \omega \right) + \\ & \left[1 - \left(1 - \cos i \right) \cos \epsilon - \cos i \right] \cos \left(\Omega + \lambda - \omega \right) + \\ & \left[1 + \left(1 + \cos i \right) \cos \epsilon + \cos i \right] \cos \left(\Omega - \lambda + \omega \right) + \\ & \left[1 - \left(1 + \cos i \right) \cos \epsilon - \cos i \right] \cos \left(\Omega - \lambda - \omega \right) + \\ & 2 \sin i \sin \epsilon \cos \left(\lambda - \omega \right) - 2 \sin i \sin \epsilon \cos \left(\lambda + \omega \right), \end{aligned} \right. \quad (9)$$

where a_{\odot} is the acceleration due to direct solar radiation pressure (about $3.6 \times 10^{-9} \text{ m s}^{-2}$, as for the LAGEOS satellites), n is the satellite mean motion, ϵ is the obliquity of the ecliptic and λ the Earth ecliptic longitude around the Sun. The periodicities of this perturbations are also characteristic of the Earth radiation pressure, i.e., the albedo effect, and of the solar Yarkovsky–Schach thermal thrust effect. These are among the largest perturbations on a satellite orbiting the Earth. Scharroo et al. (1991) proved that also an Asymmetric Reflectivity of the satellite hemispheres could be responsible of unmodelled effects on LAGEOS semimajor axis. Successively, Metris et al. (1997) and Lucchesi (2002a) considered the effects of this perturbation on LAGEOS and LAGEOS II eccentricity vector excitations and perigee rate. This perturbation (Lucchesi 2002a) is characterized by additional spectral lines with respect from those obtained with Eq. (9). In Table 2 we report the spectral lines and the periods of the main periodic contributions to the perigee rate from the above cited NGP. As we can see, our IP of about 2187 days is very close to an integer multiple of the shorter-period lines obtained. Eq. (10) gives the effect of direct solar radiation pressure on the perigee rate difference of the CSOC satellites

$$\Delta\dot{\omega}^{\text{sun}} = \frac{3a_{\odot}}{8nae} \left\{ \begin{aligned} & -2 \cos i_1 \cos \epsilon \cos \left(\Omega_1 + \lambda + \omega \right) - \\ & 2 \left[\left(1 - \cos i_1 \right) \cos \epsilon \right] \cos \left(\Omega_1 + \lambda - \omega \right) + \\ & \left[2 \left(1 + \cos i_1 \right) \cos \epsilon \right] \cos \left(\Omega_1 - \lambda + \omega \right) + \\ & -2 \cos i_1 \cos \epsilon \cos \left(\Omega_1 - \lambda - \omega \right), \end{aligned} \right. \quad (10)$$

Table 2: NGP spectral lines on the perigee rate of the proposed satellites and their periodicities (days).

Spectral line	S1	S2
$\Omega + \lambda + \omega$	725.6	243.7
$\Omega + \lambda - \omega$	728.6	244.0
$\Omega - \lambda + \omega$	244.0	728.6
$\Omega - \lambda - \omega$	243.7	725.6
$\Omega + 2\lambda + \omega$	243.0	146.2
$\Omega + 2\lambda - \omega$	243.3	146.3
$\Omega - 2\lambda + \omega$	146.3	243.3
$\Omega - 2\lambda - \omega$	146.2	243.0
$\Omega + \omega$	735.4	732.4
$\Omega - \omega$	732.4	735.4
$\lambda + \omega$	364.9	364.9
$\lambda - \omega$	365.6	365.6
$2\lambda + \omega$	182.5	182.5
$2\lambda - \omega$	182.7	182.7
ω	365,351	365,351

where we expressed the orbital elements of S2 in terms of those of S1, that is $\cos i_2 = -\cos i_1$ and $\dot{\Omega}_2 = -\dot{\Omega}_1$. Of course, Eq. (10) is valid when the CSOC satellites are in full sun-light, i.e., neglecting the shadow effects due to the Earth. We can roughly estimate the long-period effect of direct solar radiation averaging Eq. (10) over our observational period T_{obs} . We obtain

$$\langle \Delta \dot{\omega}^{\text{sun}} \rangle_{T_{\text{obs}}} \simeq \frac{1}{T_{\text{obs}}} \int_0^{T_{\text{obs}}} \Delta \dot{\omega}^{\text{sun}} dt \approx 150 \text{ mas yr}^{-1} \quad (11)$$

Concerning the perigee rate perturbations that arise from the thermal thrust effects, as well those due to the Asymmetric Reflectivity effect, they critically depend from the satellite spin axis orientation (Lucchesi 2002a). The time evolution of the spin axis of two satellites in supplementary inclination is quite different. In Figures 4 and 5 the time evolution of the cartesian components of the CSOC satellites spin axis unit-vector are shown. These plots have been obtained applying to our satellites the Bertotti and Iess (1991) spin axis evolution model in the Farinella et al. (1996) up-dated version. As we can see from the plots, the equatorial components (S_x, S_y) evolution is quite different for the two satellites, while the S_z component may be considered more or less constant, in a *first-approximation* approach, for

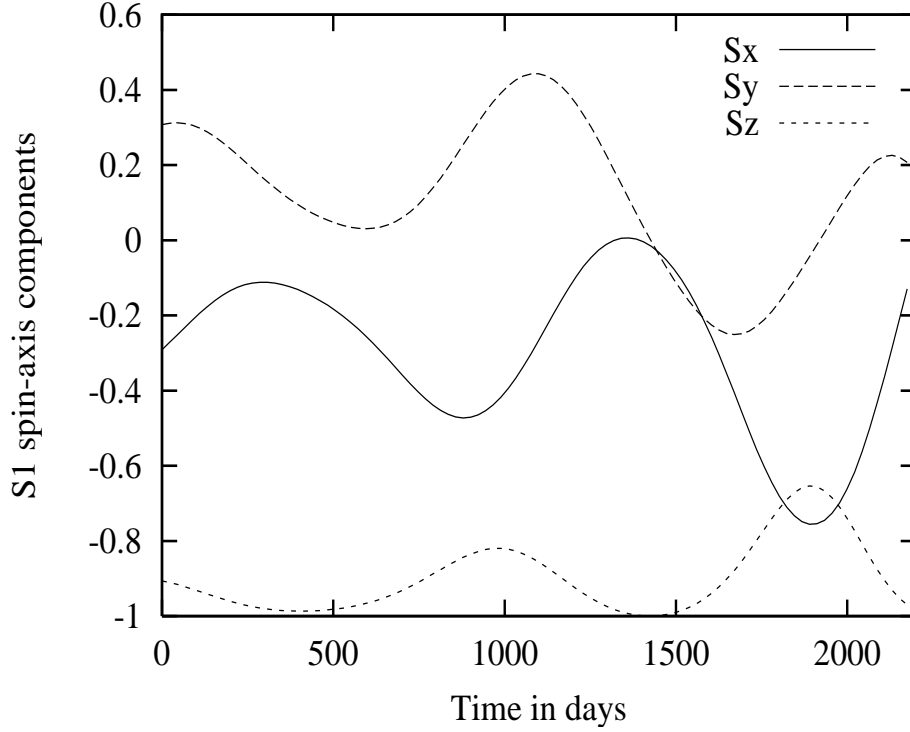


Figure 4: S1 spin axis components evolution over the IP integration time. The same magnetization parameters of LAGEOS have been adopted.

both satellites. Then, it is not so straightforwardly to compute the analytical expressions of the Thermal Thrust effects—as well as of the Asymmetric Reflectivity effect—for the CSOC satellites perigee rate difference. Indeed, the expression we can compute more easily are valid only in the case of a fixed spin axis orientation.

Of course, in the case of the terrestrial Yarkovsky–Rubincam effect (Lucchesi 2002a), we can give the expression of the secular rate in the perigee rate difference, because it depends only from the S_z components, that we can assume constant as previously evidenced. We obtain

$$\Delta\dot{\omega}^{\text{rub}}|_{\text{sec}} = \frac{A_{\text{rub}}}{4na} \cos \theta (1 - 6 \cos^2 i) (S_{z1}^2 - S_{z2}^2) \quad (12)$$

where A_{rub} is the amplitude of the perturbative effect ($\approx -7 \times 10^{-12} \text{ m s}^{-2}$, assuming the same CCR distribution of the LAGEOS satellites), while θ represents the satellite thermal lag angle, that we have assumed to be the same for the CSOC satellites and equal to that computed in the case of LAGEOS (about 55°). Computing the averages values of the satellites S_z components

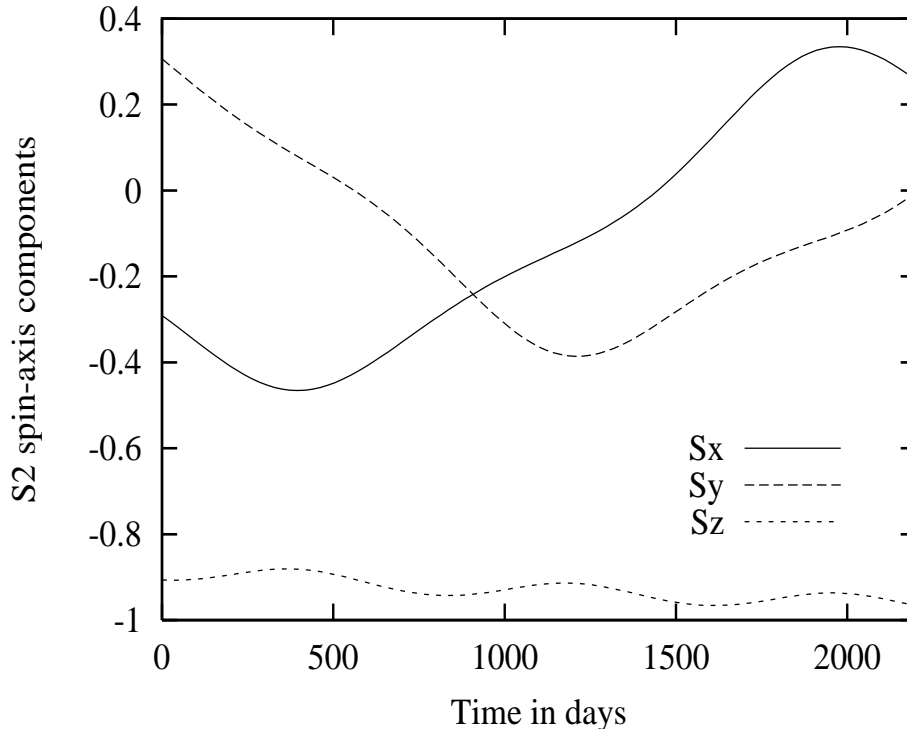


Figure 5: S2 spin axis components evolution over the IP integration time. The same magnetization parameters of LAGEOS have been adopted.

over our IP we obtain, from Eq. (12), a value of about $-0.08 \text{ mas yr}^{-1}$, indeed a negligible contribution from the secular effect with respect to the relativistic $\Delta\dot{\omega}^{\text{LT}}$ precession. However, assuming a fixed spin axis, the main contributions from the periodic terms due to the Thermal Thrust effects are the ones we have shown in Table 2, with the addition of the harmonics Ω and 2Ω in the case of the terrestrial Yarkovsky–Rubincam effect.

3.2 Numerical simulation and analysis

The orbits of the CSOC satellites have been integrated over our IP of 2187 days with a 1° step-size in the satellites eccentric anomaly. In Tables 3 and 4 we show the results we obtained for the analyzed NGP—in the satellites perigee and node—neglecting any mismodelling of the perturbative effects, i.e., their nominal amount on the elements rate. As previously pointed out, some of the analyzed NGP effects are larger than the Lense–Thirring effect, as in the case of the direct solar radiation and the Asymmetric Reflectivity in the satellites perigee rate: a

Table 3: NGP nominal effects on the perigee rate $\dot{\omega}$ (mas yr⁻¹) of the proposed satellites.

NGP	S1	S2
Solar radiation	309.86	212.68
Earth albedo	-12.86	-6.36
Earth–Yarkovsky	0.38	0.42
Solar–Yarkovsky	11.13	53.14
Asymmetric Reflectivity	201.28	206.20

Table 4: NGP nominal effects on the nodal rate $\dot{\Omega}$ (mas yr⁻¹) of the proposed satellites.

NGP	S1	S2
Solar radiation	14.77	-20.25
Earth albedo	-1.22	1.26
Earth–Yarkovsky	-0.85	0.93
Solar–Yarkovsky	-0.17	<0.01
Asymmetric Reflectivity	0.13	0.11

few hundred of mas yr⁻¹ against 44 mas yr⁻¹. In the case of the nodal rate, the direct solar radiation perturbation gives effects comparable in magnitude with the relativistic precession (about 33 mas yr⁻¹). Of course, the effect of the IP integration is evident if we compare the actual results with a 2550-day (≈ 7 -year) integration. The average long-term effects due to direct solar radiation become, respectively, about -778 mas yr⁻¹ in the case of S1 and about 3098 mas yr⁻¹ in the case of S2, i.e., more than 2.5 and 14.5 times larger! In Tables 5 and 6 we show the results for the combination of the perigee and node rates—with their errors with respect to the relativistic precession—when the uncertainties of the perturbative models are considered (the third column gives the relative uncertainty for each model). We are now able to compare the results of the numerical analysis with the estimates we obtained in sub-section 3.1 from the analytic computations. For instance, the results for the direct solar radiation pressure and the terrestrial Yarkovsky–Rubincam effect (see Table 5) are in good agreement with those previously obtained. Also the spectral analysis of the numerical integration results agrees with the lines computed analytically. In figures 6 are shown the results of the Fourier analysis in the case of direct solar radiation pressure. We have found a clear evidence of the

Table 5: NGP effects on the CSOC–satellites perigee rate difference $\Delta\dot{\omega}$ (mas yr⁻¹) and their errors (%) with respect to the Lense–Thirring effect in the perigee rates combination.

NGP	$\Delta\dot{\omega}$ (mas yr ⁻¹)	Model uncertainty %	$\frac{\delta(\Delta\dot{\omega})}{\Delta\dot{\omega}_{LT}}$ (%)
Solar radiation	97.18	0.5	0.55
Earth albedo	-6.50	10	0.74
Earth–Yarkovsky	-0.04	10	$\ll 0.01$
Solar–Yarkovsky	-42.01	10	4.8
Asymmetric Reflectivity	-4.92	20	1.1

Table 6: NGP effects on the CSOC–satellites nodal rate sum $\Sigma\dot{\Omega}$ (mas yr⁻¹) and their errors (%) with respect to the Lense–Thirring effect in the nodal rates combination.

NGP	$\Sigma\dot{\Omega}$ (mas yr ⁻¹)	Model uncertainty %	$\frac{\delta(\Sigma\dot{\Omega})}{\Sigma\dot{\Omega}_{LT}}$ (%)
Solar radiation	-5.47	0.5	0.04
Earth albedo	0.04	10	< 0.01
Earth–Yarkovsky	0.08	10	0.01
Solar–Yarkovsky	-0.01	10	$\ll 0.01$
Asymmetric Reflectivity	0.24	20	0.07

two strongest lines obtained with our analytical analysis, corresponding to the spectral lines: $\Omega + \lambda \pm \omega$ and $\Omega - \lambda \pm \omega$. Adding quadratically the errors for each kind of perturbation we are able to estimate the NGP mismodelling impact on the proposed combinations

$$\delta(\Sigma\dot{\Omega}^{\text{NGP}}) = \left\{ \sum_{\text{pert}} [\delta(\Sigma\dot{\Omega})]^2 \right\}^{1/2} \simeq 0.08\% \Sigma\dot{\Omega}^{\text{LT}} \quad (13)$$

$$\delta(\Delta\dot{\omega}^{\text{NGP}}) = \left\{ \sum_{\text{pert}} [\delta(\Delta\dot{\omega})]^2 \right\}^{1/2} \simeq 5\% \Delta\dot{\omega}^{\text{LT}} \quad (14)$$

As we can see, with the sum of the nodes we obtain a smaller impact of the NGP mismodelling on the final error budget. This is due to the negligible influence of the Yarkovsky–Schach effect on the node combination, as well as for the very small impact of the direct solar radiation and the Asymmetric Reflectivity on the proposed combination. In Table 7 we show the results for the combination proposed with Eq. (3). With this combination the larger contribution to the NGP error budget is due to the Yarkovsky–Schach effect. Adding again quadratically each

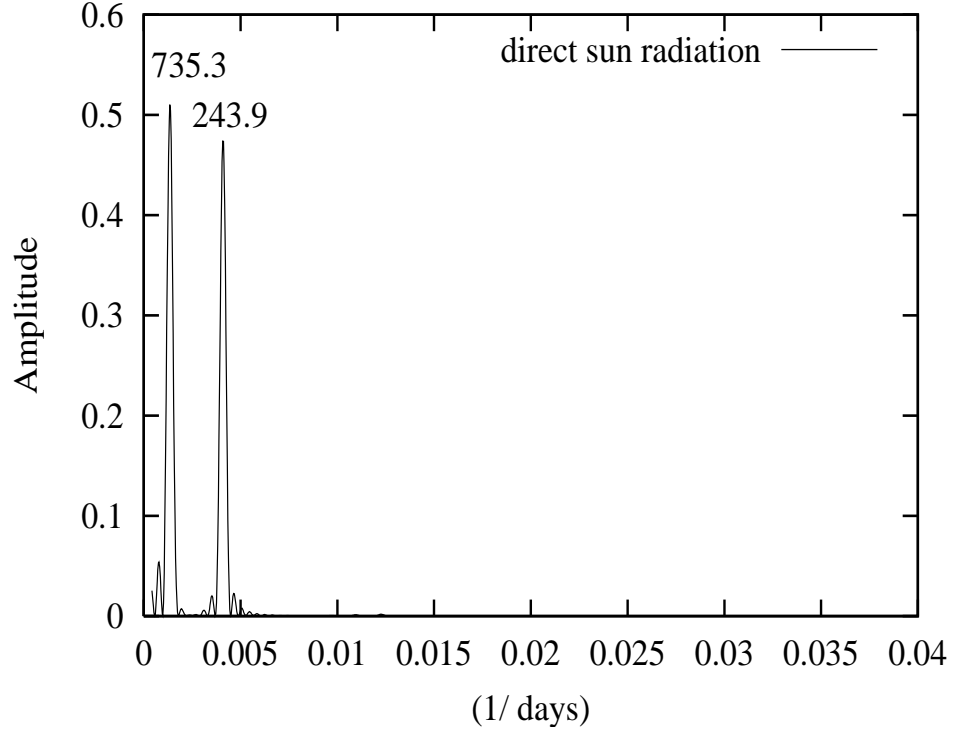


Figure 6: Fourier analysis of the perigee rate difference in the case of direct solar radiation pressure perturbation. The vertical axis gives the normalized coefficient of spectral correlation.

Table 7: NGP mismodelled effects on the CSOC-satellites nodal-rate sum and perigee-rate difference (in mas yr⁻¹) and their errors (%) with respect to the Lense-Thirring effect in the \dot{X} combination.

NGP	$\delta(\dot{X})$ (mas yr ⁻¹)	$\frac{\delta(\dot{X})}{\dot{X}_{LT}}$ (%)
Solar radiation	0.46	0.30
Earth albedo	-0.65	0.42
Earth-Yarkovsky	< 0.01	$\ll 0.01$
Solar-Yarkovsky	4.2	2.72
Asymmetric Reflectivity	-0.94	0.61

contribution of the NGP errors, we obtain

$$\delta(\dot{X}^{\text{NGP}}) = \left\{ \sum_{\text{pert}} [\delta(\dot{X})]^2 \right\}^{1/2} \simeq 2.8\% \dot{X}^{\text{LT}} \quad (15)$$

that is, a result halfway those obtained with the other combinations. The combination introduced with Eq. (3) has the advantage of a larger relativistic precession—about 154 mas yr⁻¹—with respect to the $\Sigma\dot{\Omega}$ and $\Delta\dot{\omega}$ combinations. In fact, this will be very useful when computing the integrated residuals from the SLR observations during the data-analysis. The larger slope of the \dot{X} combination residuals will indeed make a more clear evidence of the total relativistic precession \dot{X}^{LT} . We have to stress that we have followed a quite conservative error budget estimate, in particular concerning the uncertainty characterizing the Asymmetric Reflectivity effect. It is also significative to underline that actually, this very important perturbative effect is not modelled by the orbit determination programs used for the satellites data analyses. Nevertheless, it is always possible to remove some of the characteristic periodicities of this perturbation from the final fit—without affecting the slope of the Lense–Thirring effect derivation—in such a way to reduce the final rms of the plotted integrated-residuals.

4 Conclusions

In this paper we have quantitatively analyzed the scenarios offered by the proposal of launching a pair of new twin LAGEOS-like satellites in identical orbits and critical supplementary inclinations (CSOC satellites) in order to measure the gravitomagnetic Lense–Thirring effect not only by means of the sum of their nodes but also with the difference of their perigees. We have so intended to establish, on one hand, if the use of the perigees would be able to yield some benefits to the measurement of the gravitomagnetic frame dragging with respect to the sum of the nodes, and, on the other, if the launch of a new pair of SLR satellites would be justified also from the point of view of the node-only observable with respect to the LAGEOS-LARES project.

The future improvements in our knowledge of the Earth’s gravitational field thanks to the CHAMP and GRACE missions has led us to draw our attention mainly on the impact which the mismodelling of the non-gravitational perturbations (NGP) could have on the proposed gravitomagnetic observables.

The sum of the nodes would yield by far the most accurate results. The obtainable precision

should be realistically considered at the level of the order of⁴ 1%.

The difference of the perigees would be an independent, less accurate observable. It should be noticed that the practical data reduction of the perigee rates should be performed very carefully in order to account for possible, unpredictable changes in the physical properties of the satellites' surfaces which may occur after some years of their orbital life, as it seems it has happened for LAGEOS II. Such effects may yield a not negligible impact on the response to the direct solar radiation pressure. However, the great experience obtained in dealing with the perigee of LAGEOS II in the LAGEOS–LAGEOS II Lense–Thirring experiment could be fully exploited for the proposed measurement as well. The obtainable precision for the difference in the perigees should be of the order of 5%.

The combination involving the sum of the nodes with the difference of the perigees would lie at an intermediate level of accuracy⁵.

These estimates are based on the fact that, thanks to the chosen critical inclinations, over an observational time span of about 6 years (2187 days), all the time-dependent harmonic perturbations would complete some full cycles. Then, they could be viewed as empirically fitted quantities to be removed from the analyzed temporal series: this fact should yield further improvements in the error budget. Moreover, the impact of the orbital injection errors on the gravitational systematic error should be probably reduced well below 1% by the new results from CHAMP and GRACE.

Finally, we must conclude that, although appealing, the use of the alternative observable represented by the difference of the perigees of the proposed CSOC satellites would not yield any significant improvement with respect to the sum of the nodes as far as the detection of the Lense–Thirring effect is concerned. Moreover, the advantages of analyzing only the sum of the nodes of the proposed CSOC satellites with respect to the corresponding observable of the LAGEOS–LARES project would perhaps not justify the expense of the construction and the

⁴Here we do not consider the impact of measurement errors like plate motion, atmosphere and polar motion. Moreover, also the impact of the ocean tidal perturbations has not been addressed. In (Watkins *et al* 1993) six full simulations of LAGEOS–LAGEOS III data yielded a 7%–8% error.

⁵It should be considered that such results have been obtained by using the force models and the approximations which have proven to be valid for the existing LAGEOS satellites. The new satellites could be suitably built up in order to reduce the impact of the non-gravitational accelerations with respect to the existing LAGEOS satellites.

launch of such entirely new satellites, especially in view of the present-day budget restrictions of many space agencies and of the difficulties already encountered with the LARES.

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